A Mathematical Model for the Dynamic Spread of Infection caused by Poverty and Prostitution in Nigeria

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***Abstract*: In this paper, a mathematical model of (N, P_v, P_s, I, R) is presented and analyzed to gain more insight in the dynamism of Poverty and Prostitution in Nigeria. The model incorporates an infected compartment that gives a non – violent approach as intervention from the government. The stability of the system is analyzed for the existence of the Prostitute free equilibrium and existence of the Local stability of the Prostitution free equilibrium (P_sFE) was investigated via the threshold parameter [Reproduction number (R_0)] obtained using the Next generation Matrix technique. The result shows that the (P_sFE) is locally asymptotically stable at reproduction number less than unity $(R_0 < 1)$ and unstable whenever $(R_0 > 1)$. Numerical simulation was performed using reasonable sets of values from existing literatures and the results shows that high rate of government intervention will proclaim a drastic decrease to the minimum point the number of the Poverty and Prostitution population which will triggers the truncation of infection in the population.**

Keywords: **Mathematical model, Prostitution, Poverty, Local stability, Reproduction number.**

1. INTRODUCTION

The corresponding relationship between poverty and prostitution is occupation of global presence, often referred to as the world's oldest having existed for millennia [3]. Larger percent of people living in poverty therefore engage in prostitution as observed in the developing countries [8, 6].

Poverty and prostitution has an edge in contributing to rate of infection of many forms in the environment. Poverty is a complex social phenomenon whose scope travels beyond the income sufficiency. It is general phenomenon of insufficient development: in which a poor personality is in a precarious situation in which lack of abilities and opportunities of development, which is manifested in situation of low-income, malnutrition, vulnerability to disease such as (cholera, kwashiorkor, scurvy, beri-beri) to the environment inclemency, that adversely affect their opportunities of surviving and improving their good quality of life (petrol et al.). Many literatures by researchers has studied and established different pathways into prostitution which includes: homelessness, poverty, intolerance of sexual orientation, educational failure, family breakdown, unemployment, peer pressure, and delay of salaries for the government arm [7]. Our model focuses deliberately on the primary or significant effects contribution of poverty and prostitution to the spread of infection in Nigeria.

The threat posed by the influence of poverty and prostitution to the spread of infection in the population initiated and triggers this research work to develop a mathematical model which incorporates an infected compartment that allows a non-violent government intervention. The model is then use to get an insight to the dynamic spread of infection caused by poverty and prostitution and how they can be managed in the population

2. MODEL FORMULATION

The formulated model seeks to enhance non – violent government interventions in a platform which prostitution will be reasonably controlled and reduces the infections due to the poverty nature of individuals. The population is divided into five compartment namely: Non – impoverished (N), the Poverty (P_v), the Prostitution (P_s), the disease infected (I) and the Rehabilitation (R) individuals. The disease infection considered is of two bases which is the sexually transmitted disease (STD) for the prostitute individuals and diseases due to lack of good health maintenance such as balance diet as a factor of poverty. Some of the major assumption made was that a person in poverty class will conform to the prostitution class after having a contact with a prostitute over a given period of time as a form of influence. The term $\frac{\varphi P_v P_s}{T}$ represents the progression from the poverty class to the prostitution class where φ is the transmission rate. Also a rehabilitated individual may also resort to prostitution again but at a reduces rate $\frac{\delta\varphi R P_s}{T}$ where $0 \le \delta \le 1$, is the reduction fraction that account for recidivism which according to bureau of Justice Statistics (2005), this rate is around 50%. [8].

Those prostitute who have been infected with the STD immediately moves to the rehabilitation class (R) at the rate ϕ due to non – violent government intervention and the due to contact with other prostitute may revert back to prostitution at some reduce rate $\delta\varphi$ [1]. Also, the poverty individuals who have developed an infection due to lack of proper health maintenance such as balance food diet will immediately move to the rehabilitation class (R) at the rate ϕ due to non – violent government intervention. A closed population is considered.

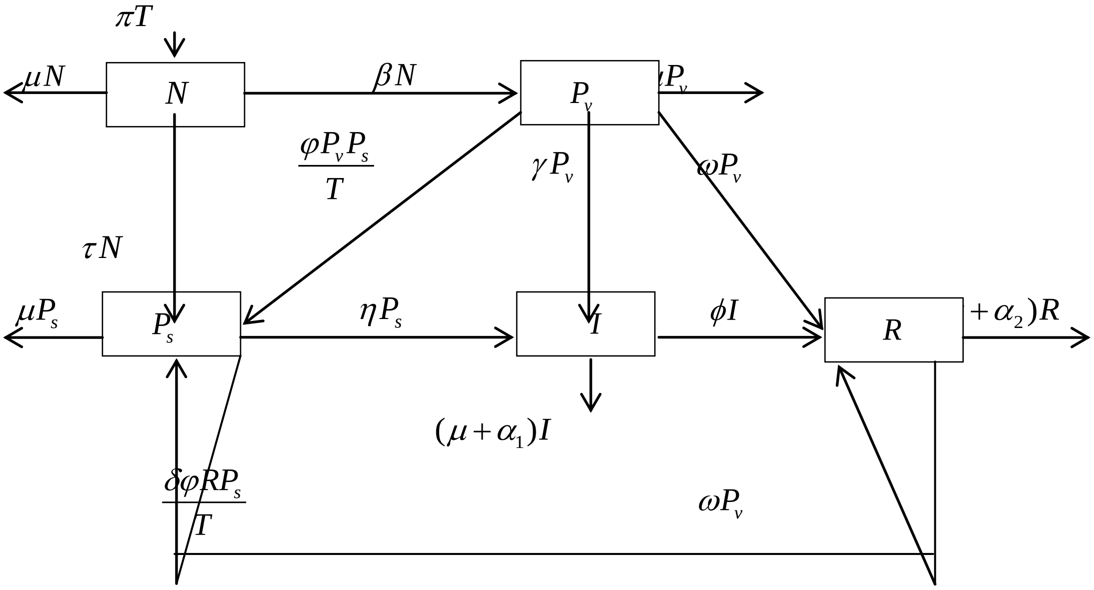

Figure 1: Schematic illustration of the Model

The govern model is given by the system of differential equations below:

$$\frac{dN}{dt} = \pi T - \beta N - \tau N - \mu N$$

$$\frac{dP_v}{dt} = \beta N - \mu P_v - \frac{\varphi P_v P_s}{T} - \omega P_v - \gamma P_v$$

$$\frac{dP_s}{dt} = \frac{\varphi P_v P_s}{T} + \frac{\delta\varphi R P_s}{T} - \mu P_s - \eta P_s - \omega P_s + \tau N \qquad (1)$$

$$\frac{dI}{dt} = \eta P_s - \mu I - \alpha_1 I - \phi I + \gamma P_v$$

$$\frac{dR}{dt} = \omega P_v + \phi I - \mu R - \alpha_2 R + \omega P_s - \frac{\delta \varphi R P_s}{T}$$

Where: $T = N + P_v + P_s + I + R$

Table 1: Description of parameter of the model

Parameters	Description
π	Recruitment rate
β	The rate of flow from the non – impoverished to the poverty individuals
ω	The conversion rate from poverty individuals to rehabilitation due to non – violent government intervention
μ	Natural death rate
φ	Progression rate at which individuals in (P_v) get into the prostitution individuals
η	Infectious rate at which prostitute get infected with any form of STD
ϕ	The rate at which infected prostitute are recruited into rehabilitation
α_1	Disease – induced death rate due to infection in the diseased infected
α_2	Disease – induced death rate due to infection in the rehabilitated individual
$\frac{\delta \varphi R P_s}{T}$	Reduced rate progression of rehabilitation back to prostitution individuals
γ	Progression rate of the poverty class to the disease infected individuals
τ	The rate of flow from the non – impoverished class to the prostitution individuals

3. ANALYSIS OF THE MODEL

A. *Prostitution free equilibrium (P_sFE):*

This is the point at which prostitution does not persist in the population, which implies setting $P_s = I = 0$ and at equilibrium, (1) is set to be equal to zero.

That is: $$\frac{dN}{dt} = \frac{dP_v}{dt} = \frac{dP_s}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = 0 \qquad (2)$$

The Prostitution free equilibrium (P_sFE) is given as: $P_s FE = \left(\frac{N_0}{T}, \frac{P_{0v}}{T}, \frac{P_{0s}}{T}, \frac{I_0}{T}, \frac{R_0}{T} \right)$

Hence, the disease free equilibrium is obtained as,

$$(S, P_v, P_s, I, R) = \left(\frac{\pi}{(\beta + \mu + \tau)}, \frac{\beta \pi}{(\beta + \mu + \tau)(\mu + \omega + \gamma)}, 0, 0, \frac{\omega \beta \pi}{(\alpha_2 + \mu)(\beta + \mu + \tau)(\mu + \omega + \gamma)} \right) \qquad (3)$$

B. *Basic Reproduction Number:*

An important notion in epidemiological models is the basic reproduction number which is usually denoted by (R_0). It measures the average number of secondary infected individual generated in his or her infectious period in the population of Susceptible [5]. It is an important parameter that tells us whether an infection will spread through the population or not. If $R_0 < 1$, then the disease dies out and spread whenever it exceeds unity i. e $(R_0 < 1)$. Using the Next generation matrix, the non – negative matrix F of the new infection terms (Transmission) and the non – singular matrix V of the other remaining transfer terms (Transition) are given by FV^{-1}:

Where:

$$F = \begin{pmatrix} \frac{\varphi P_v}{T} + \frac{\delta\varphi R}{T} & 0 \\ 0 & 0 \end{pmatrix} \qquad V = \begin{pmatrix} (\mu+\eta+\omega) & 0 \\ -\eta & (\mu+\alpha_1+\phi) \end{pmatrix} \tag{4}$$

The basic reproduction number, $R_0 = \rho(FV^{-1})$ is the spectral radius of the product FV^{-1}. Hence, for the model (2) we arrive at:

$$R_0 = \bar{R}_{P_v} + \bar{R}_R = \frac{\varphi\delta\omega\beta\pi + \varphi\beta\pi(\mu+\alpha_2)}{(\alpha_2+\mu)(\mu+\eta+\omega)(\beta+\mu+\tau)(\mu+\omega+\gamma)} \tag{5}$$

Where: $\bar{R}_{P_v} = \dfrac{\varphi\delta\omega\beta\pi}{(\alpha_2+\mu)(\mu+\eta+\omega)(\beta+\mu+\tau)(\mu+\omega+\gamma)}$

$$\bar{R}_R = \frac{\varphi\beta\pi(\mu+\alpha_2)}{(\alpha_2+\mu)(\mu+\eta+\omega)(\beta+\mu+\tau)(\mu+\omega+\gamma)}$$

$\bar{R}_{P_v}$ and $\bar{R}_R$ respectively represent the input from the Poverty class (P_v) and rehabilitation class (R). The factor $\dfrac{\omega}{(\mu+\eta+\omega)}$ and $\dfrac{\delta\varphi}{(\mu+\eta+\omega)}$ now represent the incidence rate (number of new prostitute) from the poverty class (P_v) and the rehabilitation class (R) by a single prostitute during the entire prostitution period before engaging with an STD infection. The factor $\dfrac{\beta}{(\mu+\beta+\tau)}$ represents the probability that an individual in the non – impoverish class (N) survived and entered into the poverty class, $1-\dfrac{\omega}{(\mu+\eta+\omega)}$ represents the probability that an individual in the poverty class remains in the Poverty class and $\dfrac{\omega}{(\mu+\eta+\omega)}$ represents the probability that an individual in the poverty class move into the rehabilitation class.

C. *Local Stability of the Model:*

In this section we discussed the Local stability of the model using the reproduction number obtained for the model (1).

Proposition 1: If $R_0 < 1$, then the prostitution free equilibrium is locally asymptotically stable. Otherwise, it is unstable.

Proof:

Let $f_1 = \pi T - \beta N - \tau N - \mu N$

$$f_2 = \beta N - \mu P_v - \frac{\varphi P_v P_s}{T} - \omega P_v - \gamma P_v$$

$$f_3 = \frac{\varphi P_v P_s}{T} + \frac{\delta \varphi R P_s}{T} - \mu P_s - \eta P_s - \omega P_s + \tau N \qquad (6)$$

$$f_4 = \eta P_s - \mu I - \alpha_1 I - \phi I + \gamma P_v$$

$$f_5 = \omega P_v + \phi I - \mu R - \alpha_2 R + \omega P_s - \frac{\delta \varphi R P_s}{T}$$

To obtain the Jacobian matrix, we evaluate the partial derivative of the system (6) at the prostitution free equilibrium $(P_s FE)$ states:

$$J = \begin{pmatrix} \frac{df_1}{dN} & \frac{df_1}{dP_v} & \frac{df_1}{dP_s} & \frac{df_1}{dI} & \frac{df_1}{dR} \\ \frac{df_2}{dN} & \frac{df_2}{dP_v} & \frac{df_2}{dP_s} & \frac{df_2}{dI} & \frac{df_2}{dR} \\ \frac{df_3}{dN} & \frac{df_3}{dP_v} & \frac{df_3}{dP_s} & \frac{df_3}{dI} & \frac{df_3}{dR} \\ \frac{df_4}{dN} & \frac{df_4}{dP_v} & \frac{df_4}{dP_s} & \frac{df_4}{dI} & \frac{df_4}{dR} \\ \frac{df_5}{dN} & \frac{df_5}{dP_v} & \frac{df_5}{dP_s} & \frac{df_5}{dI} & \frac{df_5}{dR} \end{pmatrix} \qquad (7)$$

Using (7) approaches, the Jacobian matrix (J) is obtained as:

$$J = \begin{pmatrix} -(\mu+\beta+\tau) & 0 & 0 & 0 & 0 \\ \beta & -(\mu+\gamma) & -\frac{\varphi P_{0v}}{T} & 0 & 0 \\ \tau & 0 & -(\mu+\eta+\omega)+\frac{\delta\varphi R_0}{T} & 0 & 0 \\ 0 & \gamma & 0 & -(\mu+\alpha_1+\phi) & 0 \\ 0 & \omega & \omega-\frac{\delta\varphi R_0}{T} & \phi & -(\mu+\alpha_2) \end{pmatrix} \qquad (8)$$

Where $\frac{P_{0v}}{T} = \frac{\beta\pi}{(\beta+\mu+\tau)(\mu+\omega+\gamma)}$ and $\frac{R_0}{T} = \frac{\omega\beta\pi}{(\alpha_2+\mu)(\beta+\mu+\tau)(\mu+\omega+\gamma)}$

The characteristics equation is obtained as $|J-\lambda I| = 0$ (9)

Where I is the 5 × 5 Identity matrix

The Eigen values of the Jacobian matrix are obtained as:

$$\lambda_1 = -(\mu+\alpha_2)$$

$$\lambda_2 = -(\mu+\beta+\tau)$$

$$\lambda_3 = -(\mu+\gamma) \qquad (10)$$

$$\lambda_4 = -(\mu+\alpha_1+\phi)$$

$$\lambda_4 = -\left\{(\mu+\eta+\omega) - \frac{\delta\varphi R_0}{T}\right\}$$

Theorem 1: The prostitution free equilibrium $(P_s FE)$ is locally asymptotically stable if $\bar{R}_R < 1$ and unstable if $\bar{R}_R > 1$.

For the system (1) to be locally asymptotically stable: $\left\{(\mu+\eta+\omega) - \frac{\delta\varphi R_0}{T}\right\} > 0$ (11)

Further simplification of (11) yields: $\left\{\frac{\delta\varphi R_0}{T(\mu+\eta+\omega)}\right\} < 1$

Where $\frac{R_0}{T} = \frac{\omega\beta\pi}{(\alpha_2+\mu)(\beta+\mu+\tau)(\mu+\omega+\gamma)}$ and thus, $\bar{R}_R < 1$

Hence, the prostitution free equilibrium $(P_s FE)$ is locally asymptotically stable.

4. NUMERICAL SIMULATION AND RESULTS

In this phase, we study numerically the expression and behaviour of the system (1) employing the parameter values compatible with [8] as given in Table 2 below and by considering the initial conditions, $T = 88875303$, $N(0) = 66656477$, $P_v(0) = 22218826$, $P_s(0) = 2221883$, $I(0) = 199970$, $R(0) = 39994$.

The numerical simulations are evaluated using the differential transformation method embedded in mathematical software (Maple 18).

Table 2: Model Parameters and Values used in Simulation

Parameters	β	ω	φ	η	ϕ	α_1	α_2	γ	τ	δ	$\pi=\mu$
Values	0.40	0.35	0.50	0.25	0.35	0.10	0.05	0.35	0.25	0.50	0.03923

The following experiments were carried out.

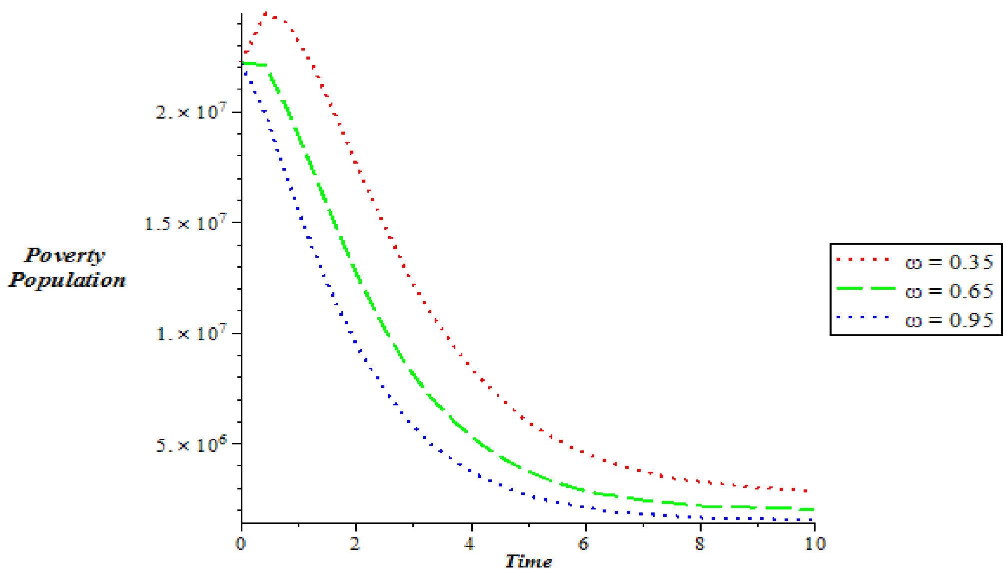

Figure 1: Effect of government intervention on the Poverty Population.

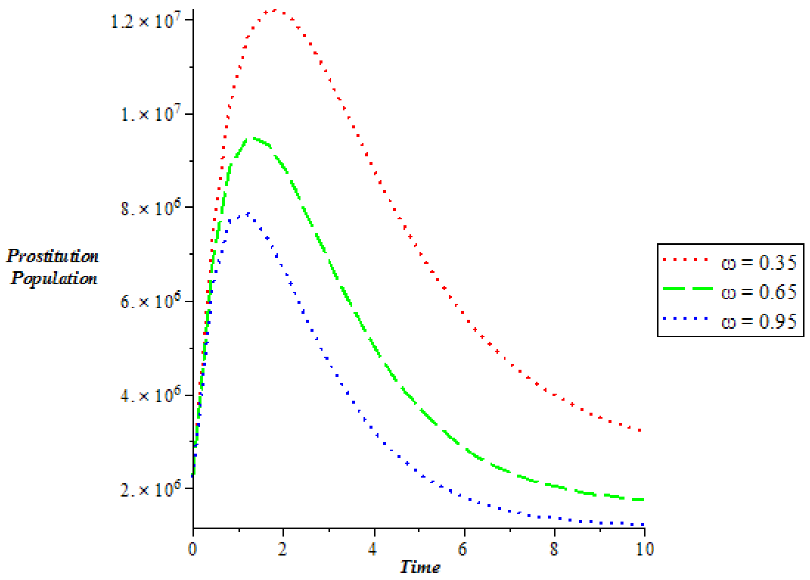

Figure 2: Effect of government intervention on the prostitution population.

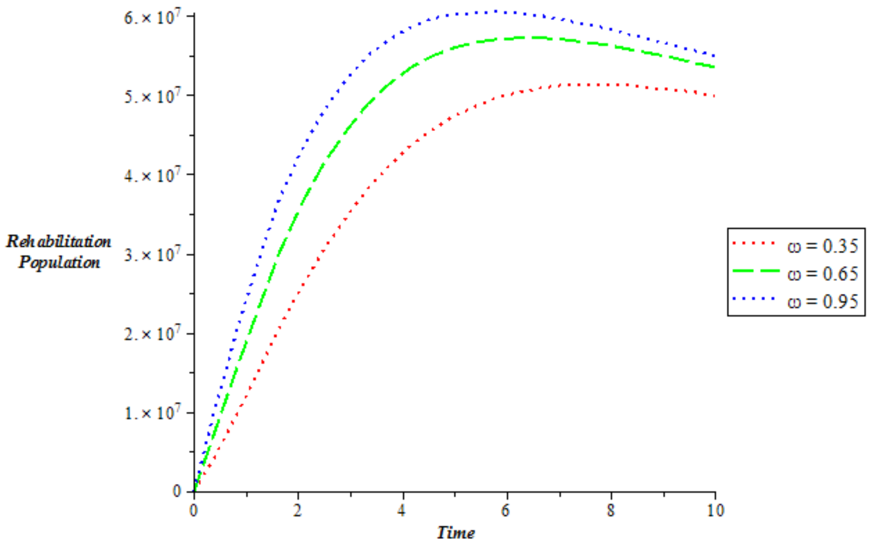

Figure 3: Effect of government intervention on the Rehabilitation Population

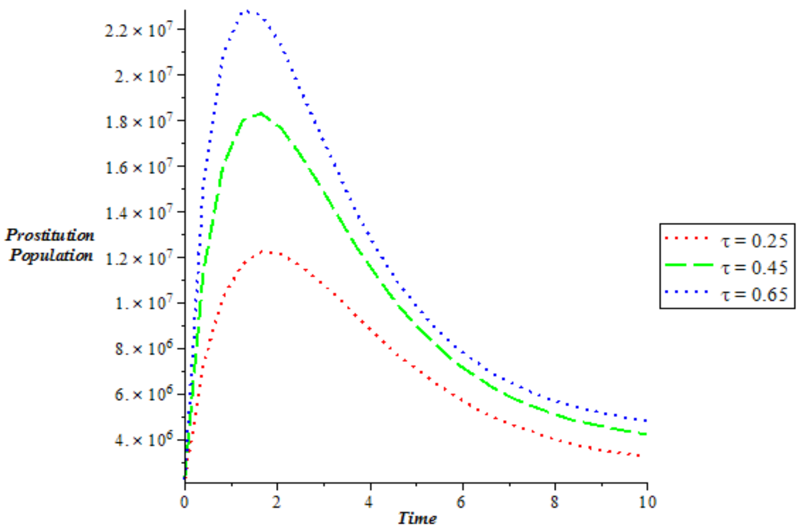

Figure 4: Effect of the flow rate of non – impoverished on the Prostitution Population.

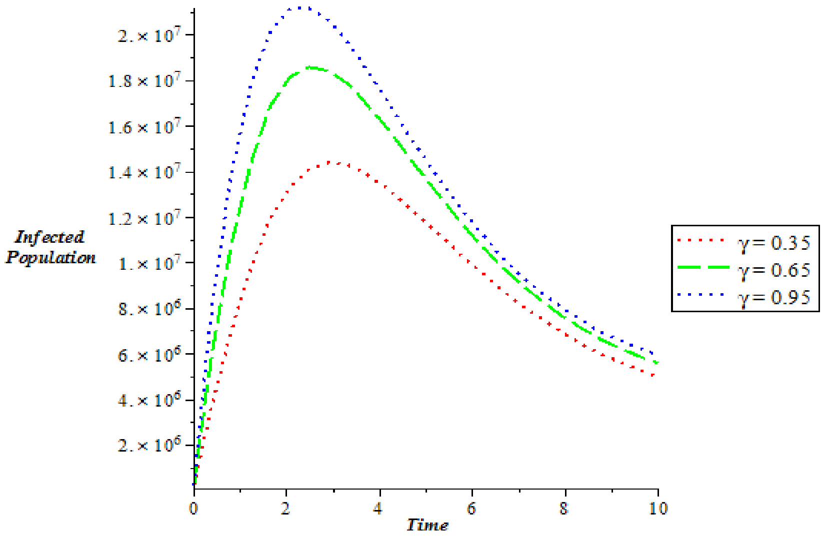

Figure 5: Effect of the progression rate of the poverty individuals on Infected Population.

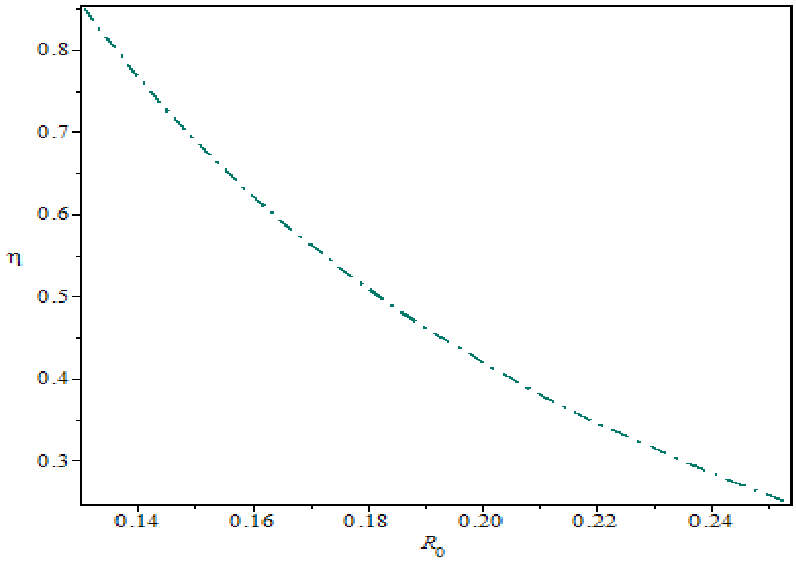

Figure 6: Effect of the infectious rate with any form of STD on Reproduction number

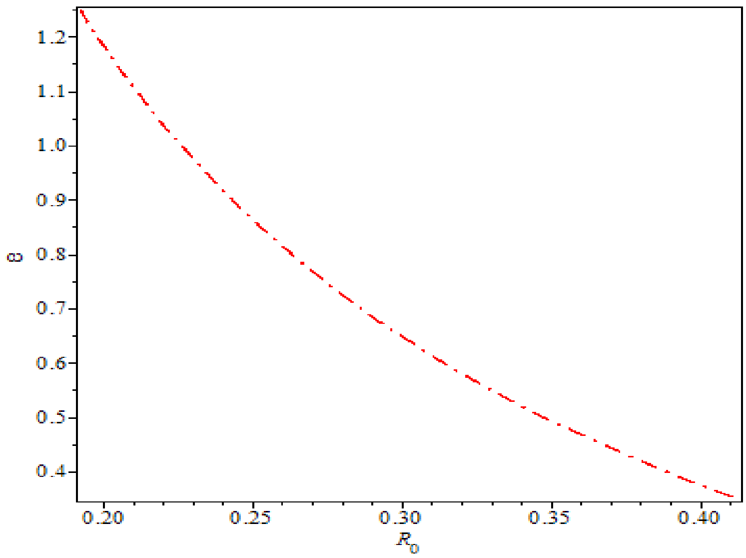

Figure 7: Effect of government intervention on the Reproduction number.

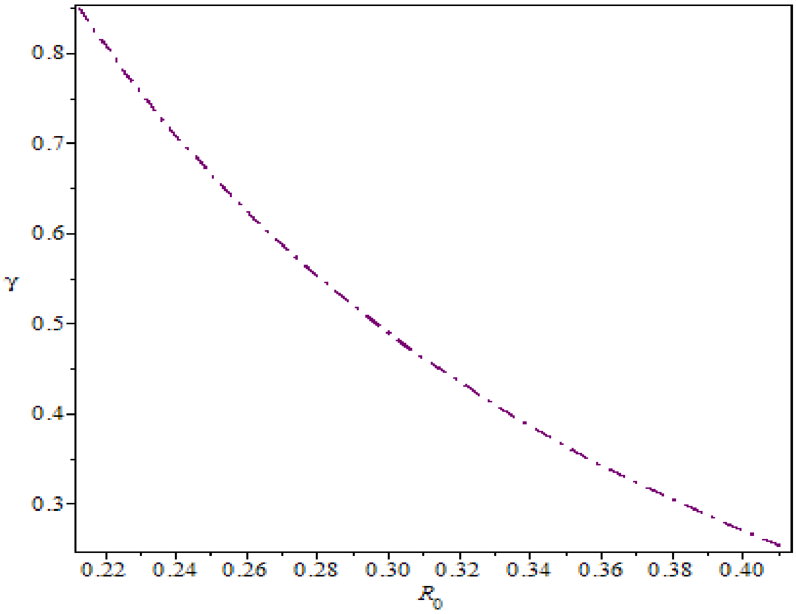

Figure 8: Effect of progression rate of poverty individuals to the infected individual on the Reproduction number.

5. DISCUSSION

Government intervention is a control necessity in decreasing the rate of poverty and prostitution in the population. Figure 1 shows the effectiveness of consistence government intervention to the eradication of poverty in the population. Consequently, this will reduce the number of individuals getting involved in prostitution as shown in figure 2 as time increases with consistence government intervention. Figure 4 shows the effect of the flow rate of non – impoverished to the prostitution population. This may be caused by factors such as peer group pressure, bad association and many more, so it is of great importance for government and parents to provide a platform like counseling, religious preaching and talks on sexual abuse to reduce to the barest minimum prostitution in the population. Figure 5 and 8 displayed the effect of progression rate of the poverty individuals to the infected population and the reproduction number respectively, and this reveals that the higher the poverty individuals in the population the higher the infected individuals. Poverty drives individuals to the level of bad health maintenance which will subsequently cause a disease to the body. Considering disease like Cholera caused due to lack of good water, it is important for government to provide an intervention to the poverty population to reduce the spread of infection in the environment. Figure 6 shows the effect of infectious rate at which prostitutes get infected with any form of STD on the reproduction number as time increases, and this shows the danger posed in the population if government intervention will not be put in place to avoid increase in prostitution to eradicate forms of STD in the population. Figure 7 shows the effect of government intervention on the reproduction number and it was observed that effective increase in government intervention reduces the reproduction number and this with time can lead to the eradication of infections in the population.

6. CONCLUSION

A five compartmental model has been presented to gain more insight into the dynamic of infections caused by poverty and prostitution in Nigeria. The result from this research has shown that effectiveness of government intervention to the poverty and prostitution population together with good parental and environment discipline to the non - impoverished population are enough to reduce to the barest minimum the spread of infection in the population.

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